Lecture 3.

Questions: <1> What is the procedure of solving h?

<2> Can we come up with simple correlations for h?

<3> Difference in h between air and water?

General analytical procedure:
heat transfer coefficient $h = -\frac{k}{T_W - T_\infty} \frac{\partial T}{\partial y}\Big|_{y=0\text{, at the wall}}$

to obtain $\frac{\partial T}{\partial y}\Big|_{y=0}$

need governing equations:

① mass conservation
② momentum conservation } → solve for velocity (u, v)

(u,v) → ③ energy conservation → solve for T distribution

This is usually complicated and time-consuming...

Recall heat conduction: only one equation (heat equation)
but covered lumped capacitance, fin, semi-infinite, multi-D...

Tricks:

<1> Similarity between momentum and energy equation.
solution $\frac{u(y)}{U_\infty} \rightarrow \Theta = \frac{T - T_W}{T_\infty - T_W}$

<2> Correlations $h = fn(?)$

$$h = f_n(k, x, \rho, C_p, \mu, U_\infty)$$

{ Still too complex, 7 variables including h
{ they all have different units.

Dimensionless analysis — Π theorem

Units	h	k	x	ρ	C_p	μ	U_∞

$$\Pi = \left[\frac{W}{m^2 K}\right]^a \left[\frac{W}{mK}\right]^b [m]^c \left[\frac{kg}{m^3}\right]^d \left[\frac{W\cdot s}{kg\cdot K}\right]^e \left[\frac{kg}{m\cdot s}\right]^f \left[\frac{m}{s}\right]^g$$

primary independent dimensions kg, m, s, W/K

7 variables, 4 dimensions $\rightarrow$ 3 Π-groups.

$\Pi = h^a k^b x^c \rho^d C_p^e \mu^f U_\infty^g$, let coefficients for [kg], [m], [s], $\left[\frac{W}{K}\right] = 0$

$$\begin{cases} \Pi_1 = \dfrac{U_\infty \rho x}{\mu} & \text{Reynolds \#, } Re_x \\ \Pi_2 = \dfrac{h \cdot x}{k} & \text{Nusselt \#, } Nu_x \\ \Pi_3 = \dfrac{C_p \mu}{k} = \dfrac{\overset{\text{momentum diffusivity}}{\nu}}{\underset{\text{thermal diffusivity}}{\alpha}} & \text{Prandtl \#, } Pr \end{cases}$$

Note: ① Note the difference between Nu and Bi

$$Nu = \frac{hx}{k_{fluid}}, \quad Bi = \frac{hx}{k_{solid}}$$

convection   conduction

② Prandtl # $= \dfrac{\nu}{\alpha} = fn\left(\dfrac{\delta}{\delta_t}\right)$

$\nu \uparrow, \ \delta \uparrow$
$\alpha \uparrow, \ \delta_t \uparrow$

When $Pr > 1, \quad \delta > \delta_t$
$Pr = 1, \quad \delta = \delta_t$
$Pr < 1, \quad \delta < \delta_t$

Gas: $Pr \sim 0.7$ (a constant on order of one)
- simple monatomic gas $Pr = \frac{2}{3}$
- diatomic gas $Pr = \frac{5}{7}$
- complex gas molecules $Pr \to 1$

air @ 300K, $Pr = 0.69$

Liquids:
① Simple molecules (H_2O, e.g.) $Pr \sim 1-10$
② liquid metal $Pr \ll 1$ (10^{-2} or less)
③ liquid with complex molecular structures, (e.g. oils) $Pr \gg 1$

Pr can vary eight orders of magnitude in common fluids

③ $\boxed{Nu_x = fn(Re_x, Pr)}$ for forced convection

got h

Recall last lecture: fluid of T_∞, U_∞ flow over a flat surface at T_w. $\nu = \alpha$

$$\Theta = \frac{T - T_w}{T_\infty - T_w} = f'(\eta) \quad \text{derivative of Blasius function}$$

$$h = 0.332\, k \frac{\sqrt{Re_x}}{x}$$

$$\boxed{Nu_x = 0.332\sqrt{Re_x}}$$

when $\alpha \neq \nu$, exact solution is :

$$\boxed{Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}} \qquad 0.6 \leq Pr \leq 50$$

Comparing ice bucket challenge (water @ 0°C) to air @ 0°C, assuming same velocity.

$$\frac{hx}{k} = 0.332\left(\frac{U_\infty x}{\nu}\right)^{1/2} Pr^{1/3}$$

	air	water
ν	12×10^{-6} [m^2/s]	1.8×10^{-6} [m^2/s]
k	0.025 [W/mK]	0.56 [W/mK]
Pr	0.7	13

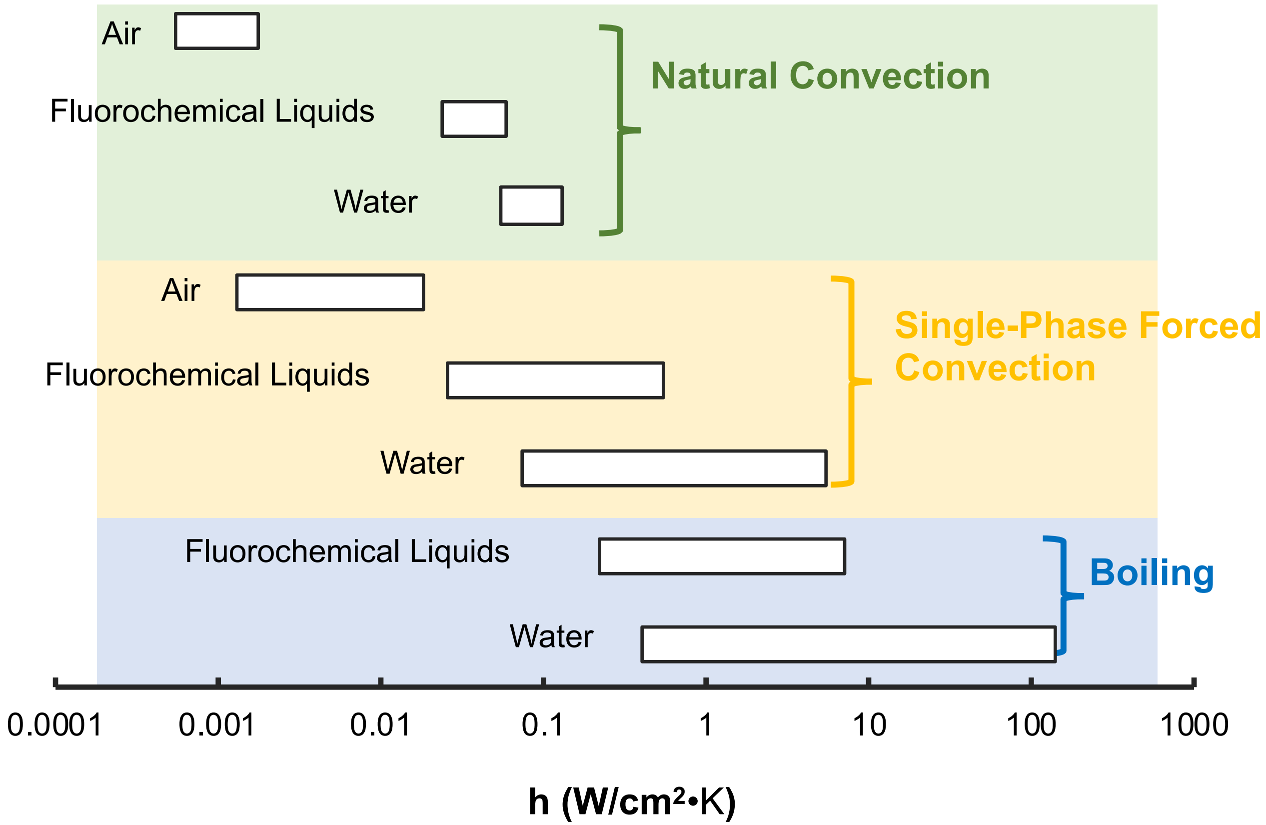
$$\frac{h_{air}}{h_{water}} \sim \frac{k_{air}}{k_{water}} \cdot \left(\frac{\nu_{air}}{\nu_{water}}\right)^{-1/2} \cdot \left(\frac{Pr_{air}}{Pr_{water}}\right)^{1/3}$$

$$\sim \frac{0.025}{0.56} \cdot \left(\frac{1.8}{12}\right)^{1/2} \left(\frac{0.7}{13}\right)^{1/3} \sim 0.0066$$

In which case do we feel colder?

Ice Bucket Challenge, water at 0 ^oC

Cold air with the same T_∞, U_∞



1 W/cm^{2}K = 10000 W/m^{2}K